المؤتمر الدولي الخامس حول
التحليل العددي والحلول المثلى

The Fifth International Conference on Numerical Analysis and Optimization

Information, Programme & Abstracts

January - كانون الثاني
6 - 9, 2020

Sponsors:

The Fifth International Conference on

NUMERICAL ANALYSIS AND OPTIMIZATION

Theory, Algorithms, Applications and Technology

Information, Programme & Abstracts

6 - 9 January, 2020

Sultan Qaboos University
Sultanate of Oman

Contents

Welcome

A warm welcome to all the participants.

We are pleased to hold this Fifth International Conference on Numerical Analysis and Optimization: Theory, Algorithms, Applications and Technology (NAOIV-2020) at the Department of Mathematics, College of Science of the Sultan Qaboos University (SQU), Muscat, Oman. This is a part of a series of conferences held at SQU once every three years. The first conference (NAO-2008) was held in April $6-8$, 2008, the second conference (NAOII-2011) was held in January $3-6$, 2011, the third conference (NAOIII-2104) was held in January $5-8$, 2014 and the fourth conference (NAOIV-2017) was held in January 2-5, 2017. We hope this conference series will become a forum where prominent scientists and experts gather to simulate the communication of innovative ideas and knowledge on new scientific methodologies. As well as that, promote scientific exchanges and discuss possibilities of further cooperation, networking and promotion of mobility of senior and young researchers and research students.

This conference will feature some of the recent research developments in theory, algorithms, and advanced applications in engineering, science and medicine to facilitate interdisciplinary research among various key sectors of pure scientific and applied knowledge. The participants will present their novel results and discuss new developments concerning applicable methods in their fields. The conference will also provide an opportunity for regional researchers to present their latest work.

A total of 19 selected invited keynote lectures and contributed papers (all peer reviewed) of the second conference (NAOII-2011) were published in the 2012 special issue of *SQU Journal for Science*, volume 17 part 1 on *Numerical Optimization* and volume 17 part 2 on *Numerical Analysis*.

A total of 13 invited keynote lectures (all peer reviewed) of the third conference (NAOIII-2014) were published in an edited book of Springer series on Proceedings in Mathematics and Statistics, *Numerical Analysis and Optimization* M. Al-Baali, L. Grandinetti and A. Purnama (Editors), Volume 134 (2015), 343p; and some contributed papers were published in *Optimization Methods and Software* and in the (special edition) *International Journal of Operational Research*, and 6 selected contributed papers in the 2015 special edited issue of *SQU Journal for Science*, volume 20(2).

Like the third conference, a total of 13 invited keynote lectures (all peer reviewed) of the fourth conference (NAOIV-2014) were published in an edited book of Springer series on Proceedings in Mathematics and Statistics, *Numerical Analysis and Optimization* M. Al-Baali, L. Grandinetti and A. Purnama (Editors), Volume 235 (2018), 304p; and some contributed papers were published in *Optimization Methods and Software* and in the (special edition) *International Journal of Operational Research*, and 7 selected contributed papers in the 2018 special edited issue of *SQU Journal for Science*, volume 23(1).

We hope you will gain much from this conference and find your involvement meaningful and worthwhile. If you have come from overseas we wish you a pleasant and enjoyable stay in Oman.

Organizing Committee

Information

- General

  The information/registration desk is located at the lobby of the Lecture Theater 1 (except on the Monday, 6 January 2020) and will be manned every day of the conference. Please ask at the desk for directions to University Mosque, Bank Muscat, post office and other facilities on campus if required.

- Presentation

  Keynote address will be 40 minutes and contributed talks will be 20 minutes. Allocated time includes discussions. To avoid delay, please contact the information desk for uploading your presentations on laptops.

- Chairing Sessions

  Please go through the programme in detail to see if you are scheduled to chair a session. Please remind the speakers to follow the programme schedule strictly.

- Refreshments

  Refreshments will be provided at the advertised times in the programme. Lunches will be served at the Faculty club.

- Conference Dinner

  The conference dinner will be held at the Crowne Plaza Muscat, on Tuesday, 7 January at 19:00. Buses will be available near the University Faculty Club. They are scheduled to depart at 18:15 and return by 22:00.

- Internet Access

  Internet access details along with username and password are given in your conference bag. Access to computers is available at the Department of Mathematics labs. Please ask at the information desk if you need assistance.

- Excursion

  For the international participants who will be staying in Oman after the conference, there will be

  (a) Muscat City Tour
  Drive past the commercial areas and arrive at the Grand Mosque. Visit the colourful Muttrah Souq. Experience the scents of exotic Arabian perfumes and spices. You will have opportunities to shop at places which specialize in famous Omani khanjars (daggers), antiques, traditional silver and gold jewelry, handicrafts in coppers, camel bones, wood, leather and hand-made Omani customs. Drive along the corniche to the old city of Muscat. Visit old Muscat, surrounded by steep mountains, and then Al Alam Palace.
  (free of charge - bus is scheduled to leave the Faculty club at 08:30 on Thursday, 9 January 2020 and the estimated time of return is 14:00)

(b) Full Day: Discover Nizwa

Drive through the spectacular Western Hajar mountains to Nizwa, which was once the capital of Oman and retains its charm as an enduring reminder of traditional Oman. Visit the 17th century round tower fort and the bustling souq known for its silver and copper handicrafts, famous for its artistic crafting. Continue to the ancient fort of Bahla with fifteen gates and 132 watchtowers. The historic town of Bahla is encircled by a winding wall, which was originally built to guard the town, also famous for its pottery.

(free of charge - bus is scheduled to leave the Faculty Club at 08:30 on Thursday, 9 January 2020 and the estimated time of return is 18:00)

Please register your name at the information desk if you are interested in any of the trips and check for any updates.

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The organising committee would like to thank the following sponsors.

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- A Mathematical Programming Language (AMPL), USA
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Abstract Titles

Invited Speaker Title	Abstract Page

Contributed Speaker Title	Abstract Page

Programme

Monday, January 6

Time	Conference Hall
8:00 - 9:00	Registration
9:00 - 10:00	Opening Ceremony

10:00 - 10:20 - Coffee / Tea Break

10:30 - Photo Session

Time	Lecture Theater 1 **Chair**: Sven Leyffer
10:40 - 11:20	Philippe Toint
11:20 - 12:00	Jànos D. Pintèr
12:00 - 12:40	Cornelis Roos

12:40 - 14:00 - Lunch Break

Time	Lecture Theater 1 **Chair**: Ekkehard Sachs
14:00 - 14:40	Michael Saunders
14:40 - 15:20	Ya-xiang Yuan
15:20 - 16:00	Michael Hintermüller

16:00 - 16:20 - Coffee / Tea Break

Time	Lecture Theater 2 **Chair**: Nezam Mahdavi-Amiri	Lecture Theater 3 **Chair**: Abul Hasan Siddiqi
16:20 - 17:00	Adil Bagirov	Pammy Manchanda
17:00 - 17:20	Ding Ma	Daniele Funaro
17:20 - 17:40	Mariam Al-Maskari	Ibtisam Al-Abri

Tuesday, January 7

Time	Lecture Theater 1 **Chair**: Jesus Sanz-Serna
9:00 - 9:40	Iain Duff
9:40 - 10:20	Francisco Facchinei
10:20 - 11:00	Sven Leyffer

11:00 - 11:20 - Coffee / Tea Break

Time	Lecture Theater 2 **Chair**: Jànos Pintèr	Lecture Theater 3 **Chair**: Philippe Toint
11:20 - 12:00	David Gay	Trond Steihaug

Time	Lecture Theater 1 **Chair**: Pammy Manchanda	Lecture Theater 2 **Chair**: Jànos D. Pintèr	Lecture Theater 3 **Chair**: Philippe Toint
12:00 - 12:20	Zouhir Mokhtari	Usama Abdelsalam	Riadh Zorgati
12:20 - 12:40	Ahmed Salam	Morteza Maleknia	Rosni Abdullah
12:40 - 13:00	Najeeb Abdulaleem	Musa Demba	Muhammed Syam

13:00 - 14:00 - Lunch Break

Time	Lecture Theater 1 **Chair**: Cornelis Roos
14:00 - 14:40	Michael Overton
14:40 - 15:20	Ekkehard Sachs
15:20 - 16:00	Dominique Orban

16:00 - 16:20 - Coffee / Tea Break

Time	Lecture Theater 1 **Chair**: Maria Teresa Vespucci	Lecture Theater 2 **Chair**: Amiya Kumar Pani	Lecture Theater 3 **Chair**: Andy Wathen
16:20 - 16:40	Issam Moghrabi	Haniffa Nasir	V. P. Ramesh
16:40 - 17:00	Ahmed Al-Siyabi	Khaled Al-Mashrafi	Palla Danumjaya
17:00 - 17:20	Amal Al-Saidi	Adel El-Sayed	Azzeddine Bellour
17:20 - 17:40	Auwal Bala Abubakar	Ahmed Al-Kasbi	Salim Al-Hudaify

Conference Dinner (buses leave at 18:15 near faculty club)

Wednesday, January 8

Time	Lecture Theater 1 **Chair**: Iain Duff
9:00 - 9:40	Desmond Higham
9:40 - 10:20	Andy Wathen
10:20 - 11:00	Jesus Sanz-Serna

11:00 - 11:20 - Coffee / Tea Break

Time	Lecture Theater 2 **Chair**: Adil Bagirov	Lecture Theater 3 **Chair**: Francisco Facchinei
11:20 - 12:00	Nezam Mahdavi-Amiri	Maria Teresa Vespucci

Time	Lecture Theater 1 **Chair**: Issam Moghrabi	Lecture Theater 2 **Chair**: Adil Bagirov	Lecture Theater 3 **Chair**: Francisco Facchinei
12:00 - 12:20	Nasser-eddine Tatar	Hassan Mohammad	Afaq Ahmad
12:20 - 12:40	Abdul Wahab	Aliyu Awwal	Medhat Awadalla
12:40 - 13:00	Abeer Al-Siyabi	Morteza Kimiaei	Hassen M. Ouakad

13:00 - 14:00 - Lunch Break

Time	Lecture Theater 1 **Chair**: Ya-xiang Yuan
14:00 - 14:40	Yu-Hong Dai

Time	Lecture Theater 2 **Chair**: Trond Steihaug	Lecture Theater 3 **Chair**: Dominique Orban
14:40 - 15:20	Amiya Kumar Pani	Abul Hasan Siddiqi

Time	Lecture Theater 1 **Chair**: Michael Overton	Lecture Theater 2 **Chair**: Trond Steihaug	Lecture Theater 3 **Chair**: Dominique Orban
15:20 - 15:40	Naveed Ahmed	Amar Oukil	Khaled Melkemi
15:40 - 16:00	Abdul-Sattar Al-Saif	Reza Farzipoor Saen	Afzal Husain
16:00 - 16:20	Bashir Zargar	Fatima Al-Raisi	Mohammad Bokhari

16:20 - 16:30 - Coffee / Tea Break

Time	Lecture Theater 1
16:30	Closing Ceremony

Thursday, January 9

8:30 -	Muscat city sightseeing and excursions to the interior of Oman

Submitted Keynote Abstracts

Acceleration and preconditioning of conjugate gradient methods

Neculai Andrei

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For improving the numerical performances of nonlinear conjugate gradient methods for unconstrained optimization two methods are considered in this talk. The first one is acceleration by modifying the stepsize of the algorithm, the second one is preconditioning. In the accelerated conjugate gradient algorithms the stepsize, computed by the Wolfe line search, is modified in a multiplicative way. Numerical examples with PRP+ and DY illustrate the efficiency and robustness of the accelerated variants of these conjugate gradient algorithms. In preconditioned conjugate gradient algorithm the search direction is modified by a preconditioning matrix. For a general nonlinear function a good preconditioner is any matrix that approximates the inverse of its Hessian. In this context the preconditioning of the modern Hager-Zhang and of the Dai-Kou conjugate gradient methods using a diagonal approximation to the Hessian as the preconditioning matrix are considered. Finally, it is shown that it must be a balance concerning the quality of the preconditioner (i.e. the closeness to the inverse Hessian), namely, if the definition of the preconditioner contains useful information about the inverse Hessian of the objective function, it is better to use the search direction corresponding to the quasi-Newton algorithm since the addition of the last term in conjugate gradient search direction may prevent it from being an efficient descent direction, unless the line search is sufficiently accurate. As an example, preconditioning the Hager-Zhang conjugate gradient algorithm using as preconditioner the self-scaling memoryless BFGS update of Perry and Shanno illustrates this behavior of the algorithms. In conclusion, preconditioning of conjugate gradient methods remain an open question with very little consensus. In contrast, acceleration by modifying the stepsize always improves the performances of conjugate gradient methods.

Piecewise linear regression: A nonsmooth optimization approach

Adil Bagirov

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In this talk, we discuss different nonsmooth optimization models of the piecewise linear regression. These models include a general nonsmooth optimization, a nonsmooth DC optimization models and also a model based on the combination of the support vector machines for regression and the nonsmooth optimization approach. Different algorithms based on the discrete gradient method and bundle methods of nonsmooth DC optimization are developed to solve the piecewise linear regression problems. We discuss the convergence of these algorithms. Their performance is demonstrated using illustrative examples. We also compare the proposed algorithms with mainstream regression algorithms using numerical results on real world data sets.

On the complexity of sequentially lifting cover inequalities for the Knapsack polytope

Yu-Hong Dai

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The well-known lifted cover inequality is widely employed in solving mixed integer programs. However, it is still an open question whether an arbitrary project lifted cover inequality can be computed in polynomial time for a given minimal cover (Gu, Nemhauser, and Savelsbergh, INFORMS J. Comput., 26: 117123, 1999). We show that this problem is NP-hard, thus giving a negative answer to the question. This is a joint work with Wei-Kun Chen.

Using shared and distributed memory in the solution of large sparse systems

Iain Duff

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We discuss the design of algorithms and codes for the solution of large sparse systems of linear equations on extreme scale computers that are characterized by having many nodes with multi-core CPUs or GPUs.

We first use two approaches to get good single node performance. For symmetric systems we use task-based algorithms based on an assembly tree representation of the factorization. We then use runtime systems for scheduling the computation on both multicore CPU nodes and GPU nodes [6]. In this work, we are also concerned with the efficient parallel implementation of the solve phase using the computed sparse factors, and we show impressive results relative to other state-of-the-art codes [3]. Our second approach was to design a new parallel threshold Markowitz algorithm [4] based on Lubys method [7] for obtaining a maximal independent set in an undirected graph. This is a significant extension since our graph model is a directed graph. We then extend the scope of both these approaches to exploit distributed memory parallelism. In the first case, we base our work on the block Cimmino algorithm [1] using the ABCD software package coded by Zenadi in Toulouse [5, 8]. The kernel for this algorithm is the direct factorization of a symmetric indefinite submatrix for which we use the above symmetric code.

To extend the unsymmetric code to distributed memory, we use the Zoltan code from Sandia [2] to partition the matrix to singly bordered block diagonal form and then use the above unsymmetric code on the blocks on the diagonal. In both cases, we illustrate the added parallelism obtained from combining the distributed memory parallelism with the high single-node performance and show that our codes out-perform other state-of-the-art codes. This work is joint with a number of people. We developed the algorithms and codes in an EU Horizon 2020 Project, called NLAFET, that finished on 30 April 2019. Coworkers in this were: Sebastien Cayrols, Jonathan Hogg, Florent Lopez, and Stojce Nakov. Collaborators in the block Cimmino part of the project were: Philippe Leleux, Daniel Ruiz, and Sukru Torun. Our codes available on the github repository `https://github.com/NLAFET`.

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Ghost penalties in nonconvex constrained optimization: Diminishing stepsizes and iteration complexity

Francisco Facchinei*

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We consider nonconvex constrained optimization problems and propose a new approach to the convergence analysis based on penalty functions. We make use of classical penalty functions in an unconventional way, in that penalty functions only enter in the theoretical analysis of convergence while the algorithm itself is penalty-free. Based on this idea, we are able to establish several new results, including the first general analysis for diminishing stepsize methods in nonconvex, constrained optimization, showing convergence to generalized stationary points, and a complexity study for SQP-type algorithms.

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* joint work with Vyacheslav Kungurtsev, Lorenzo Lampariello and Gesualdo Scutari.

Functions in AMPL

David M. Gay

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Work is well underway to augment the AMPL modeling language with functions declared in AMPL. AMPL has long allowed using functions from shared libraries, written in other languages. Functions directly expressed in AMPL should make some forms of modeling easier. In particular, providing callbacks to solvers should be easier with functions expressed directly in AMPL. Initially AMPL functions will just be available in AMPL itself, where they will enable alternate ways to constructs. Later, functions and their closures will be conveyed to solvers as part of an enhanced solver-interface facility,which will also entail some automatic differentiation machinery different from that in the current interface library

Keywords: AMPL modeling language; solver interfaces; automatic differentiation; nonlinear programming

A nonlinear spectral method for network core-periphery detection

Desmond J Higham*

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Dimension reduction is an overarching theme in data science: we enjoy finding informative patterns, features or substructures in large, complex data sets. Within the field of network science, an important problem of this nature is to identify core-periphery structure. Given a network, our task is to assign each node to either the core or periphery. Core nodes should be strongly connected across the whole network whereas peripheral nodes should be strongly connected only to core nodes. More generally, we may wish to assign a non-negative value to each node, with a larger value indicating greater 'coreness'. This type of problem is related to, but distinct from, community detection (finding clusters) and centrality assignment (finding key players), and it arises naturally in the study of networks in social science and finance. We derive and analyse a new iterative algorithm for detecting network core-periphery structure.Using techniques in nonlinear Perron-Frobenius theory we prove global convergence to the unique solution of a relaxed version of a natural discrete optimization problem. On sparse networks, the cost of each iteration scales linearly with the number of nodes, making the algorithm feasible for large-scale problems. We give an alternative interpretation of the algorithm from the perspective of maximum likelihood reordering of a new logistic core–periphery random graph model. This viewpoint also gives a new basis for quantitatively judging a core–periphery detection algorithm. We illustrate the algorithm on a range of synthetic and real networks, and show that it offers advantages over the current state-of-the-art.

* joint work with Francesco Tudisco (Gran Sasso Science Institute, Italy).

Functional-analytic and numerical issues in splitting methods for total variation-based image reconstruction

Michael Hintermüller

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Variable splitting schemes for the function space version of the image reconstruction problem with total variation regularization (TV-problem) in its primal and pre-dual formulations are considered. For the primal splitting formulation, while existence of a solution cannot be guaranteed, it is shown that quasi-minimizers of the penalized problem are asymptotically related to the solution of the original TV-problem. On the other hand, for the pre-dual formulation, a family of parametrized problems is introduced and a parameter dependent contraction of an associated fixed point iteration is established. Moreover, the theory is validated by numerical tests. Additionally, the augmented Lagrangian approach is studied, details on an implementation on a staggered grid are provided and numerical tests are shown.

Mixed-integer PDE-constrained optimization

Mirko Hahn, **Sven Leyffer**, Paul Manns, Todd Munson, and Ryan Vogt Neculai Andrei

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Many complex applications can be formulated as optimization problems constrained by partial differential equations (PDEs) with integer decision variables. This new class of problems, called mixed-integer PDE-constrained optimization (MIPDECO), must overcome the combinatorial challenge of integer decision variables combined with the numerical and computational complexity of PDE-constrained optimization. Examples of MIPDECOs include the remediation of contaminated sites and the maximization of oil recovery; the design of next-generation solar cells; the layout design of wind-farms; the design and control of gas networks; disaster recovery; and topology optimization. We will present some emerging applications of mixed-integer PDE-constrained optimization, review existing approaches to solve these problems, and highlight their computational and mathematical challenges. We show how existing methods for solving mixed-integer optimization problems can be adapted to solve this new class of problems.

A new inexact nonmonotone filter SQP algorithm

Nezam Mahdavi-Amiri and Hani Ahmadzadeh

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The class of Sequential Quadratic Programming (SQP) algorithms consists of popular methods for solving smooth constrained nonlinear optimization problems due to the warm-start capability and fast rate of local convergence. Nevertheless, the computational cost of each iteration of a conventional SQP algorithm is very high as compared to other classes of algorithms such as interior-point and augmented Lagrangian. However, SQP methods perform well on small and medium problems. In recent decades, inexact SQP algorithms were considered to enhance the performance of SQP methods for solving large problems. Here, we propose a new inexact SQP algorithm using steering and predictor directions. The iterations of the algorithm are devised as follows. In an iteration with an infeasible iterate at hand (global iteration), two directions, the so-called steering and predictor directions, are computed. The steering direction is computed as a minimizer of a linear model of the constraint violations over a trust region. This direction is used in construction of a convex feasible quadratic subproblem to admit an inexact solution, the so-called predictor direction, as a descent direction for the penalty function. The overall search direction is defined as a specified convex combination of the steering and predictor directions ensured to be descent for the constraint violations. The penalty parameter is then updated so that the search direction is also a descent direction for the ℓ_1-exact penalty function. In an iteration with a feasible iterate at hand (local iteration), a convex quadratic model is solved inexactly to yield a descent direction for the objective function. Avoiding the Maratos effect in local iterations, a nonmonotone filter strategy is employed and a local superlinear convergence is obtained. The global convergence is established under suitable assumptions concerning the global iterations. Numerical results show the proposed algorithm to be competitive.

Keywords: nonlinear optimization; sequential quadratic programming; ℓ_1-exact penalty function; filter; inexact algorithm; nonmonotone algorithm

Wavelet methods in data mining

Pammy Manchanda

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Data mining is comparatively a new interdisciplinary field developed by joint efforts of mathematicians, computer scientists and engineers. Important ingredients of data mining are (i) characteristics of different classes of data such as astronomical data, remote sensing data, financial data, meteorological data, security and surveillance data (ii) management of data (iii) reduction of size of data - dimension reduction (iv) denoising (v) data compression (vi) fusion of data (vii) enhancement of data (viii) pattern recognition in data (ix) extraction of feature in a data, similarity search (x) finding missing elements in a data (xi) prediction in a data. P. Manchanda [2] has studied application of wavelet methods in these areas. Siddiqi and Sevindir [3] have presented a wavelet based energetic approach for the analysis of electroencephalogram. Abbas and Raina [4] have reviewed 106 research papers on applications of wavelets to Data Mining. Also see [1]. There are several generalizations of wavelets, for example curvelets, shearlets [5] and wavelets constructed through Walsh functions [6]. Our research group is interested in study of replacement of wavelets by the above variants. During the talk some of those results will be presented.

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Implementing a smooth exact penalty function for nonlinear optimization

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We describe the properties of a smooth exact merit function first proposed by Fletcher (1970), and the details of our own implementation. The main computational kernel is solving structured linear systems. We show how to solve these systems efficiently by storing a single factorization per iteration when the matrices are available explicitly. We also give a factorization-free implementation. The penalty function shows particular promise when such linear systems can be solved efficiently, e.g., for PDE-constrained optimization problems where efficient preconditioners exist. Regularization provides robustness towards the solution of degenerate problems. A special feature of our implementation is the ability to evaluate inexact first and second derivatives of the merit function while preserving global convergence.

* Co-authors: Ron Estrin, Michael Saunders, Michael Friedlander.

Behavior of L-BFGS on nonsmooth optimization problems in theory and practice

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When applied to minimize non-smooth functions, the "full" BFGS method works remarkably well, typically converging linearly to Clarke stationary values, with no counter examples to this behavior known in the convex case, though its theoretical behavior is not yet well understood. In contrast, limited-memory BFGS may converge to non-stationary values, even on a simple convex function. Nonetheless, it typically reduces the objective function much faster than other methods, such as the sub-gradient method. We summarize our theoretical results and our computational experience with both full BFGS and limited memory BFGS on non-smooth optimization problems.

Keywords: non-smooth optimization; quasi-Newton methods

Global stabilization of viscous Burgers equation by nonlinear Neumann boundary feedback control: theory and finite element analysis

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Starting with objective of Computational PDEs, we plan to motivate the talk through examining some numerical experiments using a proposed finite element algorithm. In order provide some mathematical justifications as to why numbers crunched by the said algorithm make sense, we discuss some global stabilization results for the viscous Burgers equation, say, around constant steady state solution using nonlinear Neumann boundary feedback laws. The feedback laws are derived using classical approach of finding Lyapunov functional. Then, using co-conforming finite element, a priori error estimates for the state variable as well as for feedback control laws are established, which in turn repose faith on the numbers crunched by the proposed algorithm. This talk is concluded with some related problems.

Largest small n-polygons: Numerical results and estimated optima for all even values n

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The diameter of a (convex planar) polygon is defined as the maximum of the distances measured between all of its vertex pairs. LSP(n), the largest small polygon with n vertices, is the polygon of unit diameter that has maximal area $A(n)$. It has been known for almost a century that for all odd values $n = 3$, LSP(n) is the regular n-polygon. Perhaps surprisingly, the same conjecture is not valid for even values of n. Finding the polygon LSP(n) and the corresponding area A(n) for even values $n = 6$ is a long-standing challenge that leads to a class of global optimization problems. We present numerical solution estimates for all even values $6 = n = 80$, using the AMPL model development environment with the LGO global-local solver engine option. Our results are in close agreement with the results obtained by other researchers who tackled this problem using exact approaches (for $6 = n = 16$), and with the best results obtained using numerical optimization software (for selected values from the range $6 = n = 6$).

Keywords: largest small polygons; LSP optimization model; earlier solution approaches and results; AMPL model development environment; LGO solver suite for nonlinear optimization; AMPL-LGO numerical results; regression model based optimum estimates

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On some 'easy' optimization problems

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Three years ago my lecture at NAO IV was about Chubanovs method for solving homogeneous LO problems. In the analysis of a modified version of this method I had to deal with a nontrivial second-order cone optimization problem. I presented a lemma that gave a positive lower bound for the optimal value of this problem. The lemma was correct, but later on it became clear that there was a gap in its proof.

Our aim is to discuss a class of problems that can be solved in $O(n \log n)$ time. The method that we will discuss first determines the optimal partition of the problem. After this it not only yields an optimal solution, but also a certificate for its optimality. The aforementioned problem belongs to this class, as we will show.

Second order adjoints in optimization

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Second order Newton-like algorithms exhibit convergence properties superior to gradient-based or derivative-free optimization algorithms. However, deriving and computing second-order derivatives, needed for the Hessian-vector product in a Krylov iteration for the Newton step, often is not trivial. Second-order adjoints provide a systematic and efficient tool to derive such second derivative information. In this presentation we derive those in a fairly general way. We apply the developed framework to a PDE-constrained optimization problem and to machine learning for the training of neural networks.

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* joint work with Noemi Petra, University of California Merced.

The Hamiltonian Monte Carlo method and geometric integration

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Many applications, that range from statistics to physics, require the generation of samples from a probability distribution, perhaps in high dimensions. The Hamiltonian Monte Carlo method is an algorithm to obtain such samples. It originated in the physics literature and has become very popular in statistics. It is based on the numerical simulation of a system of Hamiltonian differential equations and requires that such a simulation be performed so as to exactly preserve some invariants. This talk is aimed at a general mathematical audience and will explain the interplay between probability and geometric integration, i.e., the integration of differential equations by algorithms that exactly preserve some of the geometric properties present in the system being integrated.

Computing Hamiltonian cycles in random graphs

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A Hamiltonian Cycle is a path that passes once through each node of a graph and returns to the starting node. The Hamiltonian Cycle Problem (HCP) is a special case of the Traveling Salesman Problem in that it seeks any such path, while the TSP seeks the shortest path. The HCP can be reduced to finding particular vertices of a certain polytope associated with the input graph.

Eshragh et al. (MOR 2019) implemented a simplex-type algorithm to find an HC by moving from a feasible vertex to an adjacent feasible vertex at random. To handle larger graphs, we modified the simplex algorithm in MINOS to do the same. The only change to MINOS is that Phase 2 chooses a random nonbasic variable to enter the basis. (Thus, dual variables are not required in Phase 2.)

The polytope constraints depend on a parameter beta, and the probability of finding an HC depends on beta being close to 1. In double precision we have used beta = 1 - 1e-8. A quadruple-precision version of MINOS allows beta = 1 - 1e-16 (say). We report success rates for random graphs of varying size.

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[*] joint work with Ali Eshragh, University of Newcastle.

General conformable fractional derivatives with applications

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Fractional calculus is as old as Newton-Leibnitz calculus. However its applications to real world system were in doubt for a long time. During the last decade Fractional Calculus and its applications have developed very rapidly, see for example [1] and [2].

Two new concepts of fractional derivative based on limit like classical derivative have been introduced by Khalil et al [3] and Katugompala [4], see also [5].

Both are called conformable fractional derivative. A generalization of this concept has been given by Zhao and Luo [6].

In this talk properties of general conformable fractional derivative and its applications to epilepsy and CT scan will be discussed. Relationship between generalized conformable fractional derivative and optimization will be explored.

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Twenty years of ABS methods, motivations, history and main results

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ABS is the acronym for Abaffy, Broyden and Spedicato, the three mathematicians whose interaction started the work on a unifying class of methods for solving linear and nonlinear algebraic equations and nonlinear optimization problems. The work started from a seminar of Abaffy in 1979 and went on in the framework of an international collaboration, managed mainly by Spedicato, that involved mathematicians especially from Hungary, China and Iran. We present motivations and results for the first twenty years of its development, now going on mainly via Iranian mathematicians for applications inter alia to integer problems.

We deal with the basic ideas, the unification properties and the numerical results showing that ABS methods may be more stable than classical methods. We conclude with reference to ABSMATH, a package of software developed by Bodon and Luksan, with applications to car engine optimization.

Computational science in the late 1600s up to the mid-1700s: From digit–by–digit computation to second order rate of convergence

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Computational science existed long before the computer age. Researchers in history of mathematics generally agree that Vietè's method from 1600 introduces solving (polynomial) equations in numbers in a systematic fashion using numerous examples i.e. what we today would call computational science. The method is computing the roots digit by digit and the method was refined by the English mathematicians Thomas Harriot and William Oughtred before it was picked up by Isaac Newton around 1664. Manus by Newton shows that the Newton-Raphson method was derived around 1669 and appeared in print in 1685 in the algebra book published by John Wallis. Joseph Raphson published his method in 1690. This talk covers a part of a larger project and is an overview of methods for solving nonlinear equations from 1600s up to mid-1700s. The two different implementations of the Newton-Raphson method, Newton's method as described by Wallis in 1685, Raphson's method from 1690, Halley's method from 1694 for solving nonlinear equations and the digit-by-digit methods. The methods are revisited and the differences and similarities are highlighted.

Recent results in worst-case evaluation complexity for smooth and non-smooth, exact and inexact, nonconvex optimization

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We present a review of results on the worst case complexity of minimization algorithm for nonconvex problems using potentially high degree models. Global complexity bound are presented that are valid for any models degree and any order of optimality, thereby generalizing known results for first- and second-order methods. An adaptive regularization algorithm using derivatives up to degree p will produce an eps-approximate q-th order minimizer in at most $O(eps^{(-(p+1)/(pq+1))}$ evaluations. We will also extend these results to the case of inexact objective function and derivatives.

Power generation and transmission expansion planning with high shares of renewables: A two-stage stochastic programming approach

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We consider generation and transmission expansion planning of energy systems with high penetration of intermittent renewable energy sources. Given forecasts of future values of load, fossil fuel prices and investment costs, a mixed integer linear programming model determines how the capacity mix should evolve in order to meet the demand for electricity and fulfill policy targets while minimizing the sum of operational, investment and decommissioning costs. An investment schedule is determined for the decommissioning of existing thermal plants and the construction of new generation capacity as well as of electrical regional interconnections that results in the minimum cost. Policy goals and environmental targets, such as fossil fuels and CO2 emissions reduction, are explicitly considered in the expansion plan. Since generation and transmission expansion plans are generally made for a long-term planning horizon, the system conditions are generally uncertain at the time the expansion plans are decided. Uncertainties include wind and solar build costs, emission costs and fossil fuel prices. In order to deal with the high uncertainty, in this work a two-stage stochastic programming approach is used. To accurately study the integration of large shares of renewable energy sources, a high level of temporal detail is required in this analysis. Due to the long time horizon, expansion planning problems may become computationally intractable. Most generation and transmission expansion planning models reduce computational cost by employing a rather low level of temporal and technical detail. In order to obtain high accuracy without dramatically increasing computational cost, we select a set of representative days by performing a clustering analysis on input data. By selecting representative days, the proposed approach allows maintaining a high level of temporal detail, resulting ideally suited to analyze scenarios with high penetrations of intermittent renewable energy sources.

Preconditioning for nonsymmetric Toeplitz matrices with application to time-dependent partial differential equations

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Gil Strang proposed the use of circulant matrices (and the FFT) for preconditioning symmetric Toeplitz (constant-diagonal) matrix systems in 1986 and there is now a well-developed theory which guarantees rapid convergence of the conjugate gradient method for such preconditioned positive definite symmetric systems developed by Raymond Chan, Michael Ng, Fabio Di Benedetto, Stefano Serra Capizzano and Eugene Tyrtyshnikov amongst others.In this talk we describe our recent approach which provides a preconditioned MINRES method with the same guarantees for real nonsymmetric Toeplitz systems regardless of the non-normality. We demonstrate the utility of these ideas in the context of time-dependent PDEs and show how they lead to a parallel-in-time approach. This is joint work with Elle McDonald (CSIRO, Australia), Jennifer Pestana (Strathclyde University, UK), Anthony Goddard (Durham University, UK) and Federico Danieli (Oxford University).

Efficient optimization algorithms for large-scale data analysis

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Optimization models are ubiquitous in data analysis. In this talk, we first review some efficient methods for composite convex problems, since such problems have many applications including large-scale semi-definite program problems and machine learning. We will discuss semi-smooth Newton methods, stochastic semi-smooth Newgton method, and parallel subspace correction method. We will also present algorithms for optimization problems with orthgonality constraints, including parallelizable approach for linear eigenvalut problems, parallelizable approach for nonlinear eigenvalue problems, and quasi-Newton typ method.

Accepted Contributed Abstracts

Algorithmic computation of mean time between failures (MTBF) of a solar array

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With the advancement of technologies, reduction in cost, need for sustainable development such as smart grids and smart cities around the world, the application of the Photo-Voltaic (PV) system has increased over the last past few years. A PV system uses freely available solar isolation on the earths surface to produce electricity. Solar cell is micro-level component of PV system. Several solar cells (12-128) connected in series constitute a module to increase the voltage level. Module or panel uses interchangeably in the literature. Multiple modules connected in series (name as string) and parallel constitutes a PV array. In a PV array, the series connection of modules increases the voltage level further, and the parallel connection of them increases the current level, and thus increases the power level of the PV array. Moreover, each solar cell has a bypass diode in parallel, and each module or string consists of a blocking diode that is in series with the module or the string.

The output of a PV panel depends on various factors such as solar insolation, ambient temperature, dust, cloud, rain, shadow, and wind. Moreover, cell failure, crack, and glass breakage can affect the generation of the PV panel significantly. The aforementioned factors can also impact on each and individual components such as solar cell, module, bypass diodes, blocking diodes of a solar PV panel. The failure or unavailability of a solar cell, module, bypass diode, and blocking diode can influence the output of a PV system throughout its lifetime. Such an influence can reduce components lifetime and reduce the PV output drastically, which can lead to a low rate of return on investment. Therefore, it is essential to estimate the failure rates of all the components in a PV panel. Moreover, the reliability of the PV array could be computed using the Reliability Block Diagram (RBD) concept. Since RBD has some serious limitations when networks are complex and hybrid. Hence, through this paper authors present their idea to develop an algorithmic computation framework to evaluate the Mean Time Between Failures (MTBF) of the PV components. The components level modeling and simulation for calculating MTBF of a standard PV panel is a subject to present in this research. This research can benefit the PV system owner to predict the components and system-level failure and hence, to take preventive actions such as switching to another grid or alternative source to avoid the unavailability of the PV system.

Travelling wave solutions for Fishers equation using extended homogeneous balance method

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In this work, the extended homogeneous balance method is used to derive exact solutions of nonlinear evolution equations. With the aid of symbolic computation, many new exact travelling wave solutions have been obtained for Fishers equation and Burgers-Fisher equation. Fishers equation have been widely used in studying the population for various systems, especially in biology, while Burgers-Fisher equation has many physical applications such as in gas dynamics and fluid mechanics. The used method can be applied to obtain multiple travelling wave solutions for nonlinear partial differential equations (PDEs).

Keywords: traveling wave solutions; partial differential equations; extended homogeneous balance method; Fishers equation

Sufficiency and duality for E-differentiable multiobjective programming problems involving generalized V-E-invex functions

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In our considerations, a class of E-differentiable multiobjective programming problems with both inequality and equality constraints is considered. We introduce the concepts of V-E-pseudo-invex, strictly V-E-pseudo-invex and V-E-quasi-invex functions, in which the involved functions are not necessarily differentiable, but they are E differentiable. Based upon these generalized V-E-invex functions, the sufficiency of the so-called E-Karush-Kuhn-Tucker optimality conditions are established for the considered E differentiable vector optimization problems with both inequality and equality constraints. Furthermore, the so-called vector E-dual problem in the sense of Mond-Weir and Wolfe are defined for the considered E-differentiable multiobjective programming problem and several E-duality theorems are derived also under appropriate (generalized) V-E-invexity assumptions.

Keywords: V-E-invex function; E-differentiable function; E-optimality conditions; E-Mond-Weir duality; E-Wolfe duality

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A parallel island-based genetic algorithm for inferring metabolic network

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The problem of biological network inference has become an essential problem in computational system biology. Inferring the presence or absence of links among objects involved in such networks provides a more comprehensive picture and essential information about molecular behavior for further analysis, understanding the functionality and control of complex biological networks. In our recent work, we formulated the problem of metabolic network and protein-protein interaction network inference as a multi-objective optimization method that utilizes two sources of prior knowledge: omics data and the structural properties of a scale-free network. The proposed multiobjective genetic algorithm-based method showed promising results in inferring two different types of biological networks: metabolic network and protein interaction network. However, the proposed method to reconstruct the network is time-consuming because several evaluations must be performed. GPU computing is recently revealed as a powerful high-performance way to solve large size and time-intensive problems. In this paper, we introduce a parallel island genetic algorithm, executed on Nvidia GPU, to infer metabolic network aiming at achieving faster execution time and better performance. The preliminary results showed that the parallel island model has accelerated the execution time of the multi-objective GA-based reconstruction method; the overall performance over NVIDIA Tesla K10.G2.8GB GPU card reaches a 600-fold speedup.

Keywords: biological Network Inference; genetic algorithm; optimization; island model; GPU

New three-term conjugate gradient algorithm for solving monotone nonlinear equations and signal recovery problems

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This paper presents new three-term projection algorithm for solving nonlinear monotone equations with convex constraints. The algorithm is based on the conjugate gradient method and satisfies the sufficient descent condition. Convergence of the algorithm was shown under appropriate assumptions. Numerical experiments presented show that the algorithm is competitive for solving monotone nonlinear equations. Furthermore, the proposed algorithm was applied to recover sparse signal.

Keywords: non-linear equations; nonjugate gradient; projection map; signal recovery

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An assessment of two classes of variational multiscale methods for the simulation of incompressible turbulent flows

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A numerical assessment of two classes of variational multiscale (VMS) methods for the simulation of incompressible flows is presented. Two types of residual-based VMS methods and two types of projection-based VMS methods are included in this assessment. The numerical simulations are performed at turbulent channel flow problems with various friction Reynolds numbers. It turns out that the residual-based VMS methods, in particular when used with a pair of inf-sup stable finite elements, give usually the more accurate results for second order statistics.

Evaluating wildfire management policies using stochastic dynamic model and numerical optimization

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The dramatic increase in the number of uncontrollable wildfires in the United States has become an important policy issue as they threaten valuable forest benefits, and human property. The country experienced an even longer and more intense wildfire season than normal in recent years, largely resulting from drought conditions and a buildup of flammable vegetation (fuel). Wildfires burned a total of 9.8 million acres in 2017 alone. The economic burden from wildfire reaches up to $347.8 billion annually. Thus, this study seeks to improve landowners perceptions about forest fire risk and increase their fuel management efforts.

Unlike the derived models found in previous literature, the derived stochastic dynamic model of this study is capable of determining optimal fuel treatment frequencies, timing, and level simultaneously and as a function of fire risk and fuel biomass dynamics which make the optimization problem closer to the real world situation. The landowners optimal prevention decisions depend on the type of fuel biomass growth and the association between fire arrival rate and fuel accumulation.

The model presented here represents a discrete-time Markov decision model. In such a model, an optimum policy path is a sequence of actions the manager undertakes in period t if the process is in state j to maximize the expected value of current and future net benefits. Bellmans Principle of Optimality (Bellman 1957) is used to analyze the discrete time Markov decision type of problem and expressed as follows: An optimal policy has the property that, whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. Dynamic programming is used to implement Bellmans Principle of Optimality, which has been used extensively to solve a wide variety of models (deterministic or stochastic models; discrete or continuous time, state, and action models; and models with constraints).

The derived model does not have a closed-form solution due to the specification of functional equations and endogenous variables. Therefore, this study follows Miranda and Fackler (2002) and Judd (1998) and uses a collocation method as an approximation to solve the Bellman equation. The method of collocation requires finding an approximant to the value function by a linear combination of n known basis functions defined on the state space whose coefficients are fixed in order for the value function to satisfy the Bellman equation, not at all possible states, but rather at n judiciously chosen collocation nodes (Miranda and Fackler 2002). Checking the residuals from the selected collocation method is essential to confirm that errors are minimized across the entire domain of the value function (Miranda and Fackler 2002). This study solves the Bellman equation by implementing cubic spline basis functions in MATLAB $R2017a$ using the dpsolve routine from the Miranda and Fackler (2002) COMPECON library.

Du-Fort Frankel and Lax-Wendroff schemes for one dimensional transport equation

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In this paper we test and compare numerical solutions obtained from finite difference schemes to solve the transport equation using different step sizes of time (t) and space (x). The Du Fort Frankel and Lax-Wendroff finite difference scheme are discussed. Numerical experiments were conducted using MATLAB 7.0 software. The concepts of stability of those finite difference schemes are also discussed. Our focus is on the accuracy of the scheme when applied to a complicated transport equation with discontinuous initial condition.

The present study shows that the Lax- Wendroff scheme is more stable and closer to the exact solution in comparison with the Du Fort Frankel scheme when we reduce step sizes of t and x more and more.

Spreading of brine effluents into spring-neap tidal currents: modelling and optimum conditions

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The main purpose of building large-scale desalination plants is to manage the high demand for freshwater and other various purposes; domestic, agriculture, industry. Therefore, the desalination process will become obvious and necessary to fulfill water requirements for pure water in the future. Unfortunately, desalination projects cause negative effects on the environment. Some of the most significant consequences are those associated with the marine life, energy consumption, seawater intake, and brine disposal.

Mathematically, the description of contaminants dispersion in the far-field which are discharged from a point source in shallow coastal waters is governed by a two-dimensional advection-diffusion equation. The analytical solution was obtained to describe spring-neap tidal currents and provide a more realistic description of the effects of that natural phenomena. Numerical results with graphical presentation has been conducted in the study. To figure out the role of parameters that are relevant to the spring-neap tidal currents in the mathematical model, a parametric study has been carried out. Furthermore, a parametric examination help to realize the effects of the amplitude ratio, frequency ratio and phase in order to determine the optimum conditions for reducing the potential effects of brine discharge.

Keywords: desalination plants; advection-diffusion equation; spring-neap tidal currents; optimum conditions

The mathematical model of the dynamics of bounded cartesian plumes

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A compositional plume is a fluid flow in a directional channel of finite width in another fluid of different material composition. The study of the dynamics of compositional plumes plays an essential role in many real life applications like industrial application (e.g. iron casting), environmental applications (e.g. salt fingers, and sea ice), and geophysical applications (e.g. solidification at inner core boundary (ICB) of the Earth, and mantle plumes). The dynamics of plumes have been investigated experimentally and theoretically. The experimental works observed that the plume flow seems to be stable although some experiments showed that it can be unstable. While the theoretical works showed that the plume flow is unstable. This is found to be true even if the plume is subject to rotation or/and in the presence of a magnetic field even if another plume is also present. It is noticeable that all the theoretical studies on the dynamics of these plumes are conducted in unbounded domains. The present work is to investigate theoretically the influence of vertical walls (boundaries) on the dynamics of compositional plumes in the absence/presence of rotation. The mathematical model of the dynamics of compositional plumes used the equations of conservation of mass, conservation of momentum, diffusions of heat and concentration of light material, and equation of state. It is found that the boundaries have a strong influence on the basic state solution as well as the stability of the plume, particularly when the plume is close to the boundary.

Keywords: compositional plumes; stability; bounded domain

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FEM for nonlinear subdiffusion equations with a local Lipschitz condition

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We consider the numerical approximation of a semilinear fractional order subdiffusion equation involving a Caputo derivative in time. Assuming a nonlinear source term satisfying a local Lipschitz condition, we discuss existence, stability, and provide regularity estimates for the solution of the problem. Optimal error estimates are established for a spatial discretization via piecewise linear finite elements. We further investigate fully implicit and linearized time-stepping schemes based on a convolution quadrature in time generated by the backward Euler method. Our main result provides pointwise-in-time optimal $L^2(\Omega)$-error estimates for both numerical schemes. Numerical experiments are presented, with a time-fractional Allen-Cahn equation chosen as a model, to verify the the theoretical results.

Regularization for DAG learning

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In this work, we present a new formulation for DAG learning based on a generalization of a convex regularization term. We discuss convexity of the new formulation and other properties and present empirical results using this formulation to learn graphs from data using gradient-based methods as well as simulated annealing with a large number of parameters. We present results on several evaluation metrics and discuss challenges in optimizing the proposed objectives with a number of proposed solutions. This work can be applied to solve the problem of link prediction in problem domains where DAG constraints must be satisfied. Furthermore, this work is a first step towards adjacency matrix completion for DAGs.

Computational studies of conjugate gradient methods for large-scale unconstrained optimization

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Some recent nonlinear conjugate gradient methods which are based on certain new techniques will be considered. These techniques with modifications will be used to guarantee the global convergence property of the methods for any type of line search framework. The proposed algorithms will be tested on a set of standard large-scale unconstrained problems and the numerical results will be described. It will be shown that the proposed techniques improve the performance of several conjugate gradient methods substantially.

Keywords: nonlinear conjugate gradient methods; unconstrained optimization; Wolfe conditions, backtracking line search

Analytical approximate solutions of $2D$ incompressible Navier-Stokes equations using Adomian decomposition method

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In this work, the approximate solutions of Navier-Stokes equations for two- dimensional unsteady viscous incompressible flow problems are presented. Adomian decomposition technique is used to find the new approximate analytical solutions of these flow problems. The power of this manageable method is confirmed by applying it for two selected flow problems: The first is the Taylor decaying vortices, and the second is the Kovasznay flow behind a grid, comparison its reliability and efficiency with High-order upwind compact finite-difference method is made. The convergence analysis was discussed for this method when it is applied to the flow problems, which are describing them by unsteady two-dimensional incompressible Navier-Stokes equations. Analytical approximate solutions that are obtained for two incompressible flow problems showed that the proposed method is less time consuming, quite accurate and easily implemented.

Keywords: Navier-Stokes equations; Adomian decomposition; accuracy; convergence analysis; Taylor's decay vortices; Kovasznay flow

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Quasi crystallography originating from Voronoi and Delone cells of the root lattice A_n

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Quasi crystallography is a recent phenomenon inviting new mathematical approaches. Quasi crystallographic materials discovered so far display $5-$,$8-$,$12-$ fold symmetries unlike the crystallographic materials which obey only $2-$,$3-$,$4-$,$6-$ fold symmetries. Several models have been proposed to explain the aperiodicity such as Penrose, Amman-Beenker tilings which can be constructed by two different rhombs or two different triangles. In this work we propose that all aperiodic tilings exhibiting h fold symmetry can be obtained from the projections of the root lattice A_n where $h = n + 1$ is the Coxeter number of the Coxeter-Weyl group $W(a_n)$ with n integer $n \geq 4$. We point out that the Voronoi and Delone cells of the root lattice A_n consist of identical rhombs and equilateral triangles respectively as two dimensional facets. We prove that projections of the rhombic faces onto the Coxeter plane lead to the aperiodic rhombic tilings with $h-$ fold symmetry. Similarly projections of the triangular faces of the Delone cellss exibit the $h-$ fold symmetric aperiodic tiling of the Coxeter plane by various triangles. They include the thick and thin rhombi of Penrose, Amman-Beenker tiles, Robinson triangles and Danzer triangles to name a few. It is noted that the dihedral subgroup of order $2h$ involving Coxeter element of order $h = n + 1$ of the Coxeter-Weyl group $W(a_n)$ plays a crucial role for $h-$ fold symmetric aperiodic tilings of the Coxeter plane. After setting the general scheme we give samples of patches with 4, 5, 6, 7, 8 and $12-$ fold symmetries. The work is totally based on linear algebra, group theory and computational techniques for the classification of the tiles and the figures representing the patches of symmetric tilings.

Keywords: Coxeter-Weyl group $W(a_n)$; quasicrystallography; Voronoi and Delone cells

Numerical experiments with new basic hessian approximations for large-scale nonlinear least-squares optimization

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Simple modifications to the limited memory BFGS (L-BFGS) method will be considered for solving large-scale nonlinear least-squares problem. Based upon the structure of the objective function, we suggest some matrices to define the basic starting/updating matrix instead of the usual multiple of the identity matrix. In particular, we will illustrate the way of using the Gauss-Newton Hessian, although it is not stored explicitly. Some numerical results will be described which show that the performance of the proposed modified L-BFGS methods is substantially better than that of not only the standard L-BFGS method but also of certain modified L-BFGS methods.

An evolutionary approach for data exchange optimization in parallel architectures

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Parallel architectures have obtained the popularity whether they are either fixed in their topology or more flexible in the way their architectures are constructed. These systems allow different amounts of resource sharing among its units depending on the way the units are interconnected. For one specific type of algorithms or problems, the static architectures can be designed to achieve a given requirements. The arrangement of the units of the architecture reflects the algorithm/problem sort that the system tries to tackle. However, the dynamic architectures in the other side accommodate modular and adaptable components that can be controlled using automatic software to transfer the architecture from one configuration to another to fit different kinds of algorithms/problems, and this leads to improve the performance of the whole system.

In this paper, the capabilities of the genetic algorithm as an evolutionary optimization technique have been utilized to find the optimum static structure to transfer data among processors connected together in a platform of multiprocessor. The structure should minimize the path length and maximize the possible parallel paths to ensure minimum time taken. In addition, this paper proposes different data exchange schemes, forward, virtually inverse, and hybrid, to generate dynamic configurations that enhance the system performance. Intensive experiments and qualitative comparisons have been conducted to show the effectiveness of the presented approaches. Results show significant performance improvements in terms of total data exchange time of up to 370%, 408%, 477%, and 550% when using configurations developed based on genetic algorithm, forward, virtually inverse, and hybrid data exchange techniques respectively.

A modified Perry algorithm for solving convex constrained nonlinear monotone equations and minimizing ℓ_1 regularized problem

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In this paper, a descent Perry conjugate gradient gradient for solving monotone nonlinear equation with convex constraints is proposed. The algorithm combines the projection technique and the modified Perry conjugate gradient search direction which is based on the well-known BFGS quasi-Newton direction. The convergence analysis of the algorithm is established and preliminary numerical experiments carried out show that the algorithm works well and is promising. Finally, the algorithm is applied to ℓ_1 regularized problem to recover a sparse signal from incomplete and distorted sampling measurements.

Keywords: conjugate gradient method; nonlinear monotone equations; projection method; signal recovery

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Numerical solution of high-order linear Volterra integro-differential equations by using Taylor collocation method

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The main purpose of this work is to provide a new direct numerical method for high-order linear Volterra integro-differential equations. An algorithm based on the use of Taylor polynomials is developed for the numerical solution of high-order linear Volterra integro-differential equations. It is shown that this algorithm is convergent. Numerical results are presented to prove the effectiveness of the presented algorithm

Keywords: high-order linear Volterra integro-differential equation; collocation method; Taylor polynomials; convergence analysis

Objective measures to evaluate retrieval effectiveness of news search systems for parameters of relevance and freshness

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Specifically search engines in particular and information retrieval systems in general need to be evaluated during the development process, as well as when the system is operational. This is due to the reason that the outcomes of the retrieval measure are required to be used to quantify a certain aspect of the performance of a search engine. In totality the main objective behind these evaluations is to improve the quality of the search results. This research is also an effort towards improving the quality of the search results.

Through this paper, authors propose several algorithms for evaluating retrieval effectiveness of news search engines in terms of relevance and freshness. Four Top-k merged retrieval based algorithms were developed and used to evaluate three news search engines namely- Bing news, Google news and Newslookup, on a set of 60 breaking news queries. Three algorithms were used for relevance evaluation using core information retrieval models namely- vector space model, Okapi BM25 model and Latent semantic indexing. A fourth algorithm was used solely for freshness evaluation utilizing publication date and time of the retrieved documents. We also proposed three evaluation measures for relevance and freshness in search results. As by now no single measure currently exists for evaluating both relevance and freshness of retrieved results. One obvious methodology of doing so is to measure relevance and freshness effectiveness separately and then consolidate the two measure into one. However, problems are associated with this methodology such as it is not certain that the same criteria are applied for the evaluation of both relevance and freshness. Further, if different criteria are applied will introduce bias to the evaluation. In addition, many more and richer measures exist for assessing relevance effectiveness in comparison for measures assessing freshness effectiveness, hence introduces more risk that amounts further bias.

Motivated by the above facts, we present three novel types of evaluation measures namely, Normalizes Aggregated Score (NAS@K), F-Score for relevance and freshness and Ordered Weighted Aggregation Measure (OWAM) that are designed to measure the effectiveness of both relevance and freshness in ranked lists of retrieved results. These measures were used to evaluate the same three news search engines mentioned above on the same set of 60 breaking news queries.

Mixed virtual element methods for the fourth order nonlinear parabolic problems

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Virtual element method (VEM) is a recent numerical technique which is a generalisation of the finite element method on polygons. The difference between the various finite element methods and the virtual element method is that it admits non-polynomial basis functions which are not required to be computed in practice. Instead, the degrees of freedom are chosen so that the stiffness matrix is computed exactly without explicitly knowing the basis functions. This enables us to work with more general meshes that the finite element method cannot handle in general. In this paper, we discuss mixed virtual element method (VEM) for a class of fourth order nonlinear parabolic problems. We prove some theoretical results including a priori bounds, optimal error estimates for the semi-discrete and completely discrete schemes. Finally, we perform some numerical experiments to validate the theoretical results.

Keywords: fourth order nonlinear parabolic problems; mixed virtual element method; a priori bounds; semidiscrete Galerkin method; completely discrete Galerkin method; optimal error estimates and numerical experiments

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A high order optimized explicit Runge-Kutta-Nyström method for solving periodic problems

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In this article, a high order optimized explicit Runge-Kutta-Nyström (RKN) method is constructed for solving special second order ordinary differential equations with periodic solutions. In particular, a fifth order five stage RKN method is considered in the derivation of our proposed method. To verify the order of the proposed method, we derived the local truncation error. Furthermore, the absolute stability interval of the constructed method is computed. Numerical result obtained signify the superiority of the proposed method over the existing methods in the literature.

Keywords: trigonometrically fitted; Runge-Kutta-Nyström; high order; optimized explicit; periodic problems.

Numerical solution of the fractional order advection-dispersion equation via shifted Vieta-Fibonacci polynomials

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In this article, a numerical method for the fractional order Advection-Dispersion equation (FADE) will be proposed. The fractional order derivative of the main problem will be presented using the Caputo operator of differentiation. Orthogonal polynomials of shifted Vieta-Fibonacci polynomials will be used as a basis of the desired approximate solution. The main problem will be converted into a system of ordinary differential equations. Moreover, this obtained system will be solved through the spectral collocation method and the non-standard finite difference method numerically. Some numerical applications will be presented to demonstrate the applicability and accuracy of the suggested technique.

Stable Structures in Electrodynamics

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It is possible to determine explicit solutions of the equations of electromagnetism, occupying a bounded region of the three dimensional space, topologically equivalent to a toroid. The fluid dynamics analogs of these waves are the so called vortex rings. Analytical expressions in terms of Bessel functions are known for solutions trapped inside cylindrical cavities, while in the case of rings, the use of numerical approximations is necessary. Indeed, it turns out that only a restricted range of sections are compatible with the electromagnetic constraints. Commonly, the magnetic field oscillates parallel to the major circumference of the ring and the electric field lays on the section. The configuration recalls that of a train of circulating solitons inside a rounded cavity. The shape of the ring section is studied from the numerical viewpoint, and a variety of interesting configurations is produced. This is also true in the case of spherical electromagnetic vortex rings of Hill's type, where the hole of the doughnut is a segment. In most situations, the analysis is reduced to the study of the spectrum of an elliptic operator in an appropriate bi-dimensional domain representing the section of the ring. In particular, one is concerned with determining suitable domains allowing for eigenvalues of multiplicity two (or greater). Thus, the solution process must be implemented in combination with a shape-detection algorithm. Applications concern with the development of electromagnetic whispering gallery resonators, commonly produced for a broad range of industrial applications. Typical areas of interest are in fiber telecommunications or biosensing.

Numerical analysis and performance optimization of a hybrid microchannel and jet impingement heat sink

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The recent advances in power electronics, laser technology, CPV (Concentrated Photo Voltaic), and LEDs (Light Emitting Diodes) have put forth new challenges before the thermal designers. The growing trends of ULSI (Ultra Large Scale Integration) have further increased the heat generation within these electronics. Moreover, the exponential increase in heat flux generation and the temperature has deteriorating consequences for the life span of the electronic device. The performance and efficiency of electronics are highly temperature-dependent and high temperatures may lead to irreparable losses to the device. With the air cooling techniques reaching its limits, the liquid-based microchannel cooling gained momentum during the last three decades.

A lot of research has been done to address the liquid-based microchannel cooling. The liquid based microchannel cooling provides a high heat transfer coefficient, low thermal resistance, and requires low pressure-drop and a small quantity of coolant with less overall power to drive the fluid due to its scale effects. These advantages associated with microchannel provide promising solutions to electronics cooling. The major drawback associated with microchannel cooling is temperature non-uniformity due to the unidirectional flow. The fluid enters from one end in the channel and leaves at the other end, which results in a high-temperature gradient from flow inlet to the exit of the channel. The temperature gradient is highly undesirable for electronics and has to be addressed.

The present study proposes a numerical analysis of a novel microjet impingement-effusion model for electronics cooling. The impingement microjet is surrounded by six microholes distributed hexagonally to extract the spent flow. Three-dimensional numerical analysis is performed to find out the thermal performance of the proposed model. The temperature uniformity and distribution of the heat transfer coefficient are investigated to access the temperature gradient at the heated surface.

A limited memory method for unconstrained nonlinear least squares

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In this paper, we suggest a trust region algorithm for large scale unconstrained black box least square problems. It estimates the Jacobin matrix in an adaptive subspace. Then a reduced Newton-dogleg direction is computed. Whenever the trust region step is unsuccessful, a probabilistic line search algorithm is used. If this does not improve the function value, another subspace technique based on the previous good points is tried. The numerical results show that the new algorithm is robust and efficient.

Optimal income taxation with multidimensional taxpayer types

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The optimal income taxation literature has been concentrating on economies with individuals solely differentiated by productivity since Mirrlees (1971). More realistic models should consider multidimensional heterogeneity in human nature. We propose models where people are viewed differently in up to five ways/characteristics. This project investigates scenarios where taxpayers differ in productivity, the elasticity of labor supply, basic needs, levels of distaste for work, and the elasticity of demand for consumption. In the two-dimensional cases, we have found that just one more dimension shows substantially different results. Specifically, the social welfare function reaches optimum with certain high-productivity taxpayers having negative marginal tax rates. More heterogeneous models show that income along does not well signal a taxation or subsidy policy under the goal of maximizing the total social welfare of the society. Planners may choose less social welfare redistribution in more homogeneous societies. Multidimensional optimal taxation problems are Nonlinear Optimization problems without the Linear Independence Constraint Qualification (LICQ) satisfied at each feasible point or the solution. In order to resolve the difficulty, we initially applied the state-of-the-art solver SNOPT, which has been proved to be helpful for degenerate Nonlinear Optimization Problems. We found SNOPT was not able to solve more extensive examples and were therefore motivated to develop Algorithm NCL (Nonlinearly Constrained augmented Lagrangian) and robustly solve a range of high-dimensional models. With our new method, more complex and practical taxation models can be constructed and solved in order to provide insights for the improvement of income taxation policies.

A gradient sampling method based on ideal direction for solving nonsmooth nonconvex optimization problems

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In this talk, a modification to the Gradient Sampling (GS) method for minimizing nonsmooth nonconvex functions is presented. One drawback in GS method is the need of solving a Quadratic optimization Problem (QP) at each iteration, which is time-consuming especially for large scale objectives. To resolve this difficulty, we propose a new descent direction, namely Ideal direction, for which there is no need to consider any quadratic or linear optimization subproblem. It is shown that, this direction satisfies Armijo step size condition and can be used to make a substantial reduction in the objective function. Furthermore, we prove that using Ideal directions preserves the global convergence of the GS method. Moreover, under some moderate assumptions, we present an upper bound for the number of serious iterations. Using this upper bound, we develop a different strategy to study the convergence of the method. We also demonstrate the efficiency of the proposed method using small, medium and large scale problems in our numerical experiments.

Keywords: nonsmooth and nonconvex optimization; subdifferential; steepest descent direction; gradient sampling; Armijo line search

Chebyshev approximation, non stationary wavelets and applications

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Polynomial splines are very important for many algorithms of approximation, numerical analysis and applications (signal and image processing). These families are well adapted for piece-wise smooth signals since they provide a sparse representation. However, this class is not adapted to represent some classes of signals : e.g. signals with no isolated singularities, oscillations, textures, etc. In this work, we propose a new adaptive approach based on Chebyshev spaces theory. It is a generalization of polynomials which conserves a large some of their proprieties (Taylor formula, spline construction, optimal approximation,). In order to obtain a fast algorithm, we consider invariant translation Chebyshev spaces to construct a non stationary wavelet bases algorithms. Then we determine the optimal wavelet basis (in the sense that it minimizes an error criterion) for a given function. Finally, some applications will be presented and discussed with experimental results.

A new non-secant quasi-Newton method

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The Secant equation traditionally constitutes the basis of quasi-Newton methods, as the updated Hessian approximations, at each iteration, satisfy that equation. Modified versions of the secant relation have recently been considered in the literature with encouraging outcomes. This paper continues with that idea and proceeds to derive a Secantlike modification that utilizes non-linear quantities in the construction of the Hessian (or its inverse) updating formulae. The reported results provide support as to the effectiveness of the proposed method

A multivariate spectral method for nonlinear monotone equations with applications to sparse recovery

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A spectral method in which its search direction is a product of a positive definite diagonal matrix and a negative of a residual vector is presented. The method generates a sufficient descent search direction, moreover, it is derivative-free and can solve a large-scale nonlinear monotone system of equations. Under some suitable assumptions, the global convergence results of the method are given. Furthermore, the approach has been successfully applied to recover sparse signals by transforming the ℓ_1 regularization problem into the system of nonlinear monotone equations.

Keywords: nonlinear monotone equations; diagonal spectral method; global convergence; sparse recovery; computational results.

An adapted integration method for volterra integral equation of the second kind with weakly singular kernel

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In this paper, we introduce a new product integration method in which we involve the linear interpolation to get a better approximate solution for a linear Volterra integral equation with a weakly singular kernel, figure out its effect and also we give a convergence proof. Moreover, we apply our method to a numerical example.

A unified explicit form for difference approximations of fractional and integral order derivatives

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Fractional calculus has gained a growing popularity over the last few of decades, due to its numerous interests in applications in a variety of branches in science and engineering including fractal phenomena, anomalous diffusion, viscoelasticity and biological population models.

Numerical computation with fractional differ-integral operators (FDOs), namely, fractional differential and integral operators has become an involved computational task owing to their inherent nonlocal characterization. Numerical approximations for the Riemann-Liuoville FDO have stemmed from the standard Grünwald approximation (GA) and its shifted form (SGA). Both the GA and SGA are based on a finite difference form of a desired function and their weights are obtained by means of the power series expansion of a generating function in the form of power of a linear polynomial. The GA and SGA are both of first order accuracy and the GA displays unstable solutions for certain fractional differential equations. Hence, they are not numerically effective enough. Previously published work of ours demonstrates an explicit form that yields higher order shifted Grünwald type approximations with arbitrary shifts for FDOs. The generating function in this approximation is in the form of power of a polynomial with the degree and the power, amounting to order of accuracy and order of an FDO, respectively.

In this work, we present a more general form that derives new SGA type approximations for FDOs. In this formulation, we express a fractional differential operator in terms of a d-fold differential operator and the generating functions in the form of power of a polynomial as that of the former explicit method.

An equivalent characterization for consistency and order of accuracy is established on the generating function to form a Vandermonde linear system of equations including unknown coefficients of the polynomial in the generating function. Solving this system, we obtained the coefficients in the ratio of two functions, namely, one for the numerator part and the other for the denominator part of the explicit expression. This explicitness was implemented in the preceding numerator-denominator form to compute the coefficients in exact fractional form enabling efficient computation of fractional derivatives.

We used the explicit method to approximate the Riemann-Liuoville FDOs and numerical results were recorded for base differential order 2. We further noted some super convergence points of our explicit form.

Keywords: fractional derivative; Grünwald approximation; generating functions; fractional differential equations

Efficient numerical approaches for the nonlinear nonlocal behavior of electrically actuated carbon nanotube resonators

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In the past few decades, problem formulation based on the Classical Continuum Mechanics (CCM) permitted to develop powerful, robust and reliable simulation tools to solve complex problems in the field of structural mechanics of micro and nano-eletromechanical systems (MEMS and NEMS). However, it is well known that at the molecular level, the mater is somehow discrete and heterogeneous, and therefore the hypotheses of the CCM, recognized as size-independent, are no longer valid in the small-scale. The CCM resulting governing equations lack an internal size dependent length scale, therefore it cannot predict any size effect and may fail when effects like size-dependency and scaling of mechanical phenomena play a crucial role, certainly do in the nano-scale. The above problems could be addressed using discrete models but all of them require a great computational effort. This fact provides a motivation towards developing modified and generalized continuum mechanics theories that are capable to capture the size effects through introducing intrinsic lengths in their respective formulation. Within this category fall the classical couple stress theory and the strain gradient theory, started in 1960s with the works of Mindlin and Tiersten. Another size-dependent continuum theory which contains only one material length scale parameter is the nonlocal continuum mechanics initiated by Eringen and coworkers back in 1972, which has been widely used to analyze many localized problems, such as wave propagation, dislocation, and crack singularities.

A different approach to non-local mechanics has been recently introduced in the context of fractional calculus. Fractional calculus (FC) is a branch of mathematical analysis that studies the differential operators of an arbitrary order. The attractiveness of FC application lays in the fact that: (1) fractional differential operators are nonlocal, and (2) there are many definitions of fractional derivatives. In the last decades fractional differential equations have been used to capture physical phenomena in the nano-scale that cannot be caught by classical differential models. This talk will discuss some of the ongoing theoretical research of electrically actuated carbon nanotubes (CNTs) based NEMS resonators, where the fractional continuum mechanics (FCM) approach will be utilized to modal their respective nonlocal structural behavior. The nano-resonator static, eigenvalue problem (natural frequencies and modal shapes), and dynamic responses are obtained and the effects of the length-scale parameter are discussed and contrasted with those obtained with the solutions derived from the CCM. The presented model provides a basis for the study of the linear and nonlinear structural behaviour of elastic nanostructures showing significant nonlocal length/scale effects.

A robust ranking approach based on the value system embedded under DEA cross-efficiency

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Under a data envelopment analysis (DEA) framework, full ranking of a group of decision making units (DMUs) can be carried out through an adequate amalgamation of the crossefficiency (CE) scores produced for each DMU. In this paper, we propose a ranking procedure that is based on amalgamating the weight profiles selected over the cross-evaluation rather than related CE scores. The new approach builds, for each DMU, a collective weight profile (CWP) by exploiting the preference voting system embedded within the matrix of weights, which views the assessing DMUs as voters and the input/output factors as candidates. The occurrence of zero votes is discussed as a special case and a two-level aggregation procedure is developed. The CWPs that are produced extend the concept of collective appreciation to the input/output factors of each DMU so that group dynamics is truly reflected, mainly in decision making circumstances where factor prioritization is necessary for making choices or allocating resources. Moreover, these CWPs exhibit almost no zero weights without any a priori restriction on the weight profiles. The robustness of the proposed ranking approach is evaluated on a set of manufacturing systems.

Keywords: data envelopment analysis; collective value system; weight profile; crossefficiency; preference voting; ranking

A priori mesh refinement strategy for singularly perturbed turning point problem

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In this talk, we present a priori mesh refinement strategy for a class of singularly perturbed turning point problem with interior layer. We present the design and analysis of the proposed algorithm and we prove that the algorithm is parameter uniformly convergent. We also present a few numerical experiments to demonstrate the efficiency of the algorithm on capturing the interior layer.

Keywords: singular perturbation; turning point; interior layer

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How to assess sustainability of supply chains by fuzzy Malmquist productivity index?

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Sustainable supply chain management is one of the important topics in the transportation industry. This paper proposes a fuzzy Malmquist productivity index (FMPI) using both optimistic and pessimistic frontiers in data envelopment analysis (DEA). FMPI is the first attempt that appears in this paper. The FMPI reflects the productivity changes of decision making units (DMUs) over time. The proposed FMPI is used to assess sustainability of suppliers. A case study shows efficacy of the proposed model.

Keywords: Sustainability; intercity freight transportation; data envelopment analysis (DEA); fuzzy Malmquist productivity index (FMPI); double frontiers DEA

On theoretical and numerical aspects of symplectic reductions of a matrix to upper J-Hessenberg form

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The reduction of a matrix to a J-Hessenberg form is needed in structure preserving methods for a class of large and sparse structured matrices, in the context of computations of eignevalues and eignevectors of such matrices. The reduction of such matrices, to a condensed form, the upper J-Hessenberg form, is based on symplectic similarity transformations. The algorithm $JHESS$, or the recent $JHMSH$ algorithm and its variants, are the main algorithms used for such reduction. This reduction is a crucial step in the SR-algorithm (which is a QR-like algorithm), structure-preserving, for computing eigenvalues and vectors, of this class of structured matrices.

Unlike its equivalent in the Euclidean case, these algorithms may meet fatal breakdowns, causing brutal stops of the computations or encounter near-breakdowns, which are source of serious numerical instability.

In this talk, give necessary and sufficient condition under which breakdowns occur. Then, we present efficient strategies for curing them. We derive also strategies for curing near-breakdowns. The effectiveness of such strategies are illustrated by numerical experiments.

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Optimization of one step block methods hybrid points for solving first-order ordinary differential equations

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Some optimized one-step hybrid block methods for the numerical solution of first-order initial value problems are presented. The methods take into consideration one, two, and three off-step hybrid points which are chosen appropriately to optimize the local truncation errors of the main formulas for the block. We proved that these methods are zero-stable and consistent. Some numerical examples are discussed to show the efficiency and the accuracy of the proposed method.

The main challenges in studying fractional viscoelastic problems

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In this talk I will discuss the main difficulties encountered in investigating fractional versions of viscoelastic problems. It will be explained why the ideas used in the integer (second order) case are no longer valid in the fractional case. We will start by the well-posedness question and suggest some ways how to establish it. For the stability, several difficulties arise from the setting of the problems themselves. Besides, terms which act as dampers have no longer the dissipation property in the fractional case. Again, we suggest a way for the setting of the problems and how to prove stability for such cases.

A model based joint sparsity approach for inverse elastic medium scattering

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In this talk, I will focus on an inverse problem arising in Magnetic Resonance Elastography. A compressive sensing based algorithm will be presented for efficient and accurate reconstruction of the spatial support and material parameters of multiple inhomogeneous elastic inclusions in a bounded elastic material. Only a few measurements of the elastic displacement field over a very coarse grid (in the sense of Nyquist sampling rate) will be taken into account, on contrary to classical algorithms assuming continuous or dense grid measurements. Our proposed algorithm is not only very accurate but also computationally efficient as it does not require any linearization or iterative proceedure. The breakthrough comes from a novel interpretation of Lippmann-Schwinger type integral representation of the displacement field in terms of unknown densities (connected to the displacement and strain fields inside the inclusions) having common and sparse spatial support on the location of inclusions. Accordingly, the support identification problem is recast as a joint sparse recovery problem that renders such densities and the support of the inclusions simultaneously. Then, using the leverage of the learned densities and associated fields inside the inclusions, a linear inverse problem for quantitative evaluation of material parameters is formulated. The resulting problem is then converted to a noise robust constraint optimization problem. For numerical implementation, modified Multiple Sparse Bayesian Learning (M-SBL) algorithm and the Constrained Split Augmented Lagrangian Shrinkage Algorithm (C-SALSA) are used due to their robustness with respect to noise. The efficacy of the proposed framework will be manifested through a variety of numerical examples. The significance of this investigation is due to its pertinence for bio-medical imaging and non-destructive testing, wherein the real physical measurements are only available on a sub-sampled coarse grid [1]. The proposed algorithm is the first one tailored for parameter reconstruction problems in elastic media using highly under-sampled data.

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Bernstein type inequalities for polynomials

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Let $P(z) = \sum_{j=0}^{n} a_j z^j$ be a polynomial of degree n and $P'(z)$ be its derivative.

$$\underline{Max}_{|z|=1}|P'(z)| \leq nMax_{|z|=1}|P(z)| \tag{1}$$

$$\underline{Max}_{|z|=R>1}|P(z)| \leq R^n Max_{|z|=1}|P(z)| \tag{2}$$

Inequality (1) is a famous result due to S. Bernsteins whereas inequality (2) is a simple deduction from maximum modulus principle. If we restrict ourselves to a class of polynomials having no zero in $|z| < 1$, then both the inequalities (1) and (2) are sharpened. In fact, in this case Paul Erdos conjectured that if $P(z)$ is a polynomial of degree n and does not vanish in $|z| < 1$, then

$$\underline{Max}_{|z|=1}|P'(z)| \leq \frac{n}{2} Max_{|z|=1}|P(z)| \tag{3}$$

Li, Mohapatra used rational functions and generalised Inequality (3).

In this talk I shall present some refinements and generalisations by using rational functions. Besides some recent developments in approximation theory will be discussed.

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Solving ill-conditioned linear systems with a preconditionning stochastic matrice-based approach

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In quite diverse application areas, we have to deal with the very bad conditioning of the matrix of a linear system of equations, making its resolution difficult to handle numerically. This issue appears in optimization for example when solving linear programs with interior point algorithms, linear systems at each iteration becoming more and more poorly conditioned as we approach the optimal solution. Devising efficient solution approaches for such system of equations is therefore of practical interest.

For dealing with such an issue, we propose an efficient approach based on $\mathcal{L}_1$ norm and stochastic matrices. It consists first in preconditioning the initial system with an approximation of a generalized inverse of a matrix, supposed non-negative, such that the preconditioned matrix is stochastic. This basic approach, which allows us to retrieve in an original way the Schultz-Hotelling-Bodewigs algorithm of iterative refinement of the approximate inverse of a matrix, is then extended to hermitian, semi-definite positive matrices and finally generalized to any complex rectangular matrices. We present the spectral analysis of the proposed preconditionning matrix, which generalizes the Cimminos matrix. We show that the convergence condition requested for iteratively solving linear systems is always satisfied. Thanks to its properties, this matrix can then be efficiently used in different solving schemes like preconditioned conjugate gradients, Richardson or generalized Schultz-Hotelling-Bodewig (SHB) of any order p. We show that SHB scheme of order p converges to an inverse generalized solution, which is characterized, following Tanabe theory. Regarding numerical results obtained on some pathological test-cases, some of the proposed algorithms are empirically shown to be more efficient on ill-conditioned problems and more robust to error propagation than most known classical techniques we have tested (Gauss, Moore-Penrose inverse, minimum residue, conjugate gradients, Kaczmarz, Cimmino).

We end on a very early prospective application of this stochastic matrices based approach, aiming at computing some parameters such as the extreme values, the mean, the variance, of the solution of a linear system prior to its resolution. Such an approach, if it were to be efficient, would be a source of information on the solution of a system of linear equations.

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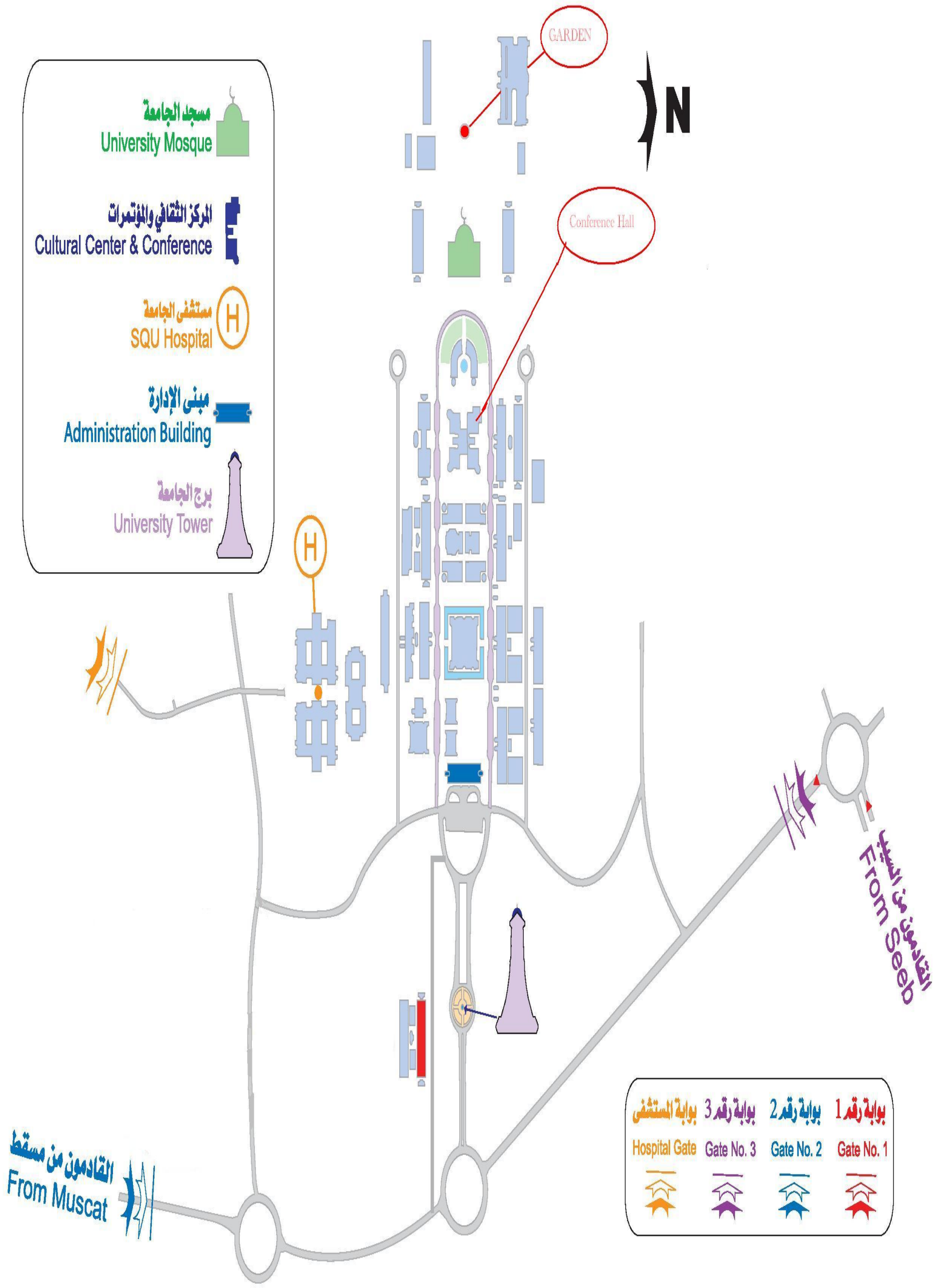
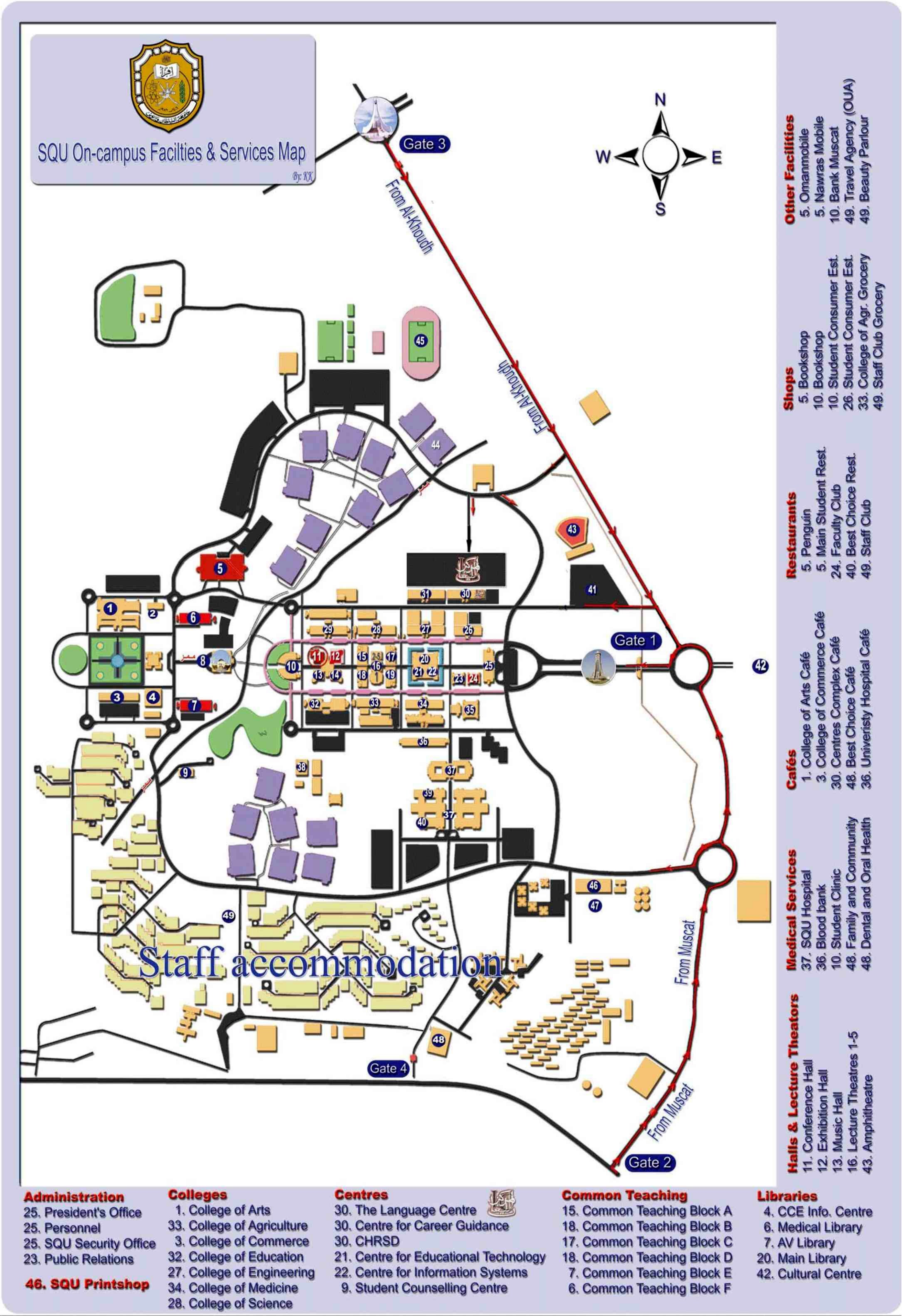


GARDEN
N
Conference Hall
مسجد الجامعة
University Mosque
المركز الثقافي والمؤتمرات
Cultural Center & Conference
مستشفى الجامعة
SQU Hospital
مبنى الإدارة
Administration Building
برج الجامعة
University Tower
H
القادمون من السيب
From Seeb
القادمون من مسقط
From Muscat
بوابة المستشفى
Hospital Gate
بوابة رقم 3
Gate No. 3
بوابة رقم 2
Gate No. 2
بوابة رقم 1
Gate No. 1



SQU On-campus Facilties & Services Map
By: KK
Gate 3
From Al-Khoudh
From Al-Khoudh
Gate 1
Gate 2
Gate 4
From Muscat
From Muscat
Staff accommodation
N
S
W
E
Other Facilities
5. Omanmobile
5. Nawras Mobile
10. Bank Muscat
49. Travel Agency (OUA)
49. Beauty Parlour
Shops
5. Bookshop
10. Bookshop
10. Student Consumer Est.
26. Student Consumer Est.
33. College of Agr. Grocery
49. Staff Club Grocery
Restaurants
5. Penguin
5. Main Student Rest.
24. Faculty Club
40. Best Choice Rest.
49. Staff Club
Cafés
1. College of Arts Café
3. College of Commerce Café
30. Centres Complex Café
48. Best Choice Café
36. Univeristy Hospital Café
Medical Services
37. SQU Hospital
36. Blood bank
10. Student Clinic
48. Family and Community
48. Dental and Oral Health
Halls & Lecture Theators
11. Conference Hall
12. Exhibition Hall
13. Music Hall
16. Lecture Theatres 1-5
43. Amphitheatre
Administration
25. President's Office
25. Personnel
25. SQU Security Office
23. Public Relations
46. SQU Printshop
Colleges
1. College of Arts
33. College of Agriculture
3. College of Commerce
32. College of Education
27. College of Engineering
34. College of Medicine
28. College of Science
Centres
30. The Language Centre
30. Centre for Career Guidance
30. CHRSD
21. Centre for Educational Technology
22. Centre for Information Systems
9. Student Counselling Centre
Common Teaching
15. Common Teaching Block A
18. Common Teaching Block B
17. Common Teaching Block C
18. Common Teaching Block D
7. Common Teaching Block E
6. Common Teaching Block F
Libraries
4. CCE Info. Centre
6. Medical Library
7. AV Library
20. Main Library
42. Cultural Centre


SQU ج س ق
NAO V
Muscat
مسقط
2020
جامعة السلطان قابوس
Sultan Qaboos University
المؤتمر الدولي الخامس حول التحليل العددي والحلول المثلى
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